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Identifying and addressing common fraction misconceptions

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Identifying and addressing common fraction misconceptions

- Fractions are one of the most challenging mathematical topics to teach and learn.
- Many difficulties arise when students rely on the same strategies they used successfully to solve whole number tasks to solve fraction tasks.
- Misconceptions may not be identified as students can get the correct answers using 'faulty' fractional thinking using their whole number strategies.
- Fractions can be represented in different ways e.g. as part of a length or an area, part of a quantity or as a position on a number line. For some students, changes in representations adds another layer of difficulty.

Curriculum expectations

Year 1: Recognise and describe one-half as one of two equal parts of a whole (VCMNA091)

Year 2: Recognise and interpret common uses of halves, quarters and eighths of shapes and collections (VCMNA110)

Year 3: Model and represent unit fractions including 1/2, 1/4, 1/3, 1/5 and their multiples to a complete whole (VCMNA136)

Year 4

- Investigate equivalent fractions used in contexts (VCMNA157)
- Count by quarters, halves and thirds, including with mixed numerals. Locate and represent these fractions on a number line (VCMNA158)

Curriculum expectations

Year 5: Compare and order common unit fractions and locate and represent them on a number line (VCMNA187)

- Investigate strategies to solve problems involving addition and subtraction of fractions with the same denominator (VCMNA188)

Year 6:

- Compare fractions with related denominators and locate and represent them on a number line (VCMNA211)

- Solve problems involving addition and subtraction of fractions with the same or related denominators (VCMNA212)

- Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies (VCMNA213)

Why are fractions so difficult?

Common misconceptions?

Finding a fractional part of a whole

1. What is one-half of 6 counters?
2. What is one-third of 6 counters?
3. What is two-thirds of 6 counters?
4. What is one-quarter of 8 counters?
5. What is three-quarters of 8 counters?
6. What is three-fifths of 10 counters.

Finding a fractional part of a whole

Think About:

Three-quarters of 8 is 6 and three-fifths of 10 is also 6.

How can two fractional parts give the same answer?

Finding the whole given the part

- If 5 counters represents one-half of the whole group how many counters in the whole group?
- If 3 counters represents one-quarter of a group how many counters in the group?
- If 4 counters represents two-fifths of the group how many counters in the whole group?
- If 21 counters represents seven-sixths of the group how many counters in the whole group?

Discussion: Why do students (and adults) find these tasks more difficult than the previous tasks?

Linear model: Fraction Strips

For this activity you will need **one** paper strip (20 cm long)

The linear model: Fraction Strips

- Fold one strip into halves

- Fold the halves in half (quarters)

- Fold the quarters in half (eighths).

Linear Model: Fraction Wall

Use your Fraction Strip to mark the Fraction Wall (and label with unit fractions)

Linear Model: Fraction Wall

Use your Fraction Strip to mark the Fraction Wall (and label with unit fractions)

one whole							
$1/2$				$1/2$			
$1/4$		$1/4$		$1/4$		$1/4$	
$1/8$	$1/8$	$1/8$	$1/8$	$1/8$	$1/8$	$1/8$	$1/8$

What is the problem?

This is incorrect:

one whole							
$^1/_2$				$^2/_2$			
$^1/_4$		$^2/_4$		$^3/_4$		$^4/_4$	
$^1/_8$	$^2/_8$	$^3/_8$	$^4/_8$	$^5/_8$	$^6/_8$	$^7/_8$	$^8/_8$

Using the Fraction Wall

1. Find fractions which are equivalent to:

$\frac{1}{4}$ $\frac{3}{4}$

2. Find fractions which are smaller than:

$\frac{1}{2}$ $\frac{3}{4}$

3. Find two fractions which add up to $\frac{3}{4}$

4. Which is larger $\frac{1}{2}$ or $\frac{3}{4}$? How do you know?

Discussion

How do Fraction Walls help students model fractional relationships:

- fraction sizes
- equivalent fractions
- basic addition, subtraction, multiplication and division of fractions?

Linear model: Number lines

For this activity you need number lines the same length as your paper strips.

Number line Activity 1

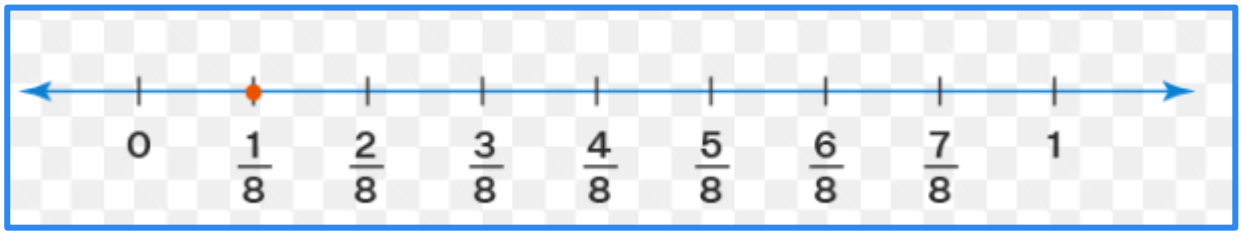
Mark the ends of **one** number line 0 and 1.

Place the following fractions as accurately as you can:

$$\frac{1}{2} \quad \frac{1}{4} \quad \frac{3}{4} \quad \frac{7}{8} \quad \frac{1}{8} \quad \frac{3}{8}$$

How can you check whether these fractions are in the correct spots on the number line?

Number line marked in eighths



Remember:

- one-half is equivalent to four-eighths
- one-quarter is equivalent to two-eighths
- three-quarters is equivalent to six-eighths

Number line Activity 2

Mark the ends of a number line 0 and 2.

Place the following fractions as accurately as you can:

$\frac{1}{2}$ $\frac{8}{8}$ $\frac{6}{4}$ $\frac{5}{8}$ $\frac{12}{8}$ $\frac{8}{4}$ $\frac{15}{8}$

Discussion:

How will you check the accuracy of these fractions?

Number Line Activities

Discussion:

- What fractional concepts are being developed and/or emphasised through these activities?
- What questions could you ask students as they place the fractions on the number line to draw out their mathematical thinking and/or to further emphasise the fractional concepts?
- How could you make this activity easier for students?
- How could you make this activity more challenging for students?

Which is larger?

$\frac{3}{5}$ or $\frac{2}{3}$

$\frac{3}{5}$ or $\frac{5}{8}$

$\frac{3}{5}$ or $\frac{3}{4}$

What is this student 'thinking'?

When Sam was asked to compare $^{3}/_{5}$ and $^{5}/_{8}$ he said:

"Three-fifths is larger because there is less of a gap between the three and the five than the five and the eight (in the second fraction)."

We call this gap thinking.

Which is larger: $\frac{3}{5}$ or $\frac{2}{3}$?

misconception	correct	incorrect
larger numerator		
smaller numerator		
larger denominator		
smaller denominator		
smaller sum or product		
larger sum or product		
larger gap		
smaller gap		

Which is larger: $\frac{3}{5}$ or $\frac{5}{8}$?

misconception	correct	incorrect
larger numerator		
smaller numerator		
larger denominator		
smaller denominator		
smaller sum or product		
larger sum or product		
larger gap		
smaller gap		

Which is larger: $\frac{3}{5}$ or $\frac{3}{4}$?

misconception	correct	incorrect
larger numerator		
smaller numerator		
larger denominator		
smaller denominator		
smaller sum or product		
larger sum or product		
larger gap		
smaller gap		

Which is larger?

misconception	$\frac{3}{5}$ or $\frac{2}{3}$	$\frac{3}{5}$ or $\frac{5}{8}$	$\frac{3}{5}$ or $\frac{3}{4}$
larger numerator		yes	
smaller numerator	yes		
larger denominator		yes	yes
smaller denominator	yes		
smaller sum or product	yes		
larger sum or product		yes	yes
larger gap		yes	
smaller gap	yes		yes

Comparison of fractions

Year 5: Compare and order common unit fractions and locate and represent them on a number line (VCMNA187)

Year 6: Compare fractions with related denominators and locate and represent them on a number line (VCMNA211)

Year 7: Compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line (VCMNA242)

Comparing Fractions Diagnostic Test

http://www.smartvic.com/teacher/mdc/number/N35007P.html

For each pair of fractions, either CIRCLE the larger fraction, OR write = between them

$\frac{5}{8}$	$\frac{1}{3}$
$\frac{2}{3}$	$\frac{3}{4}$
$\frac{4}{6}$	$\frac{2}{3}$
$\frac{4}{5}$	$\frac{3}{8}$
$\frac{5}{7}$	$\frac{3}{8}$
$\frac{3}{10}$	$\frac{2}{5}$

$\frac{1}{4}$	$\frac{3}{4}$
$\frac{3}{5}$	$\frac{2}{5}$
$\frac{1}{3}$	$\frac{1}{4}$
$\frac{3}{5}$	$\frac{3}{4}$
$\frac{1}{6}$	$\frac{2}{12}$
$\frac{7}{8}$	$\frac{11}{12}$

Calculations with fractions

Year 5: Investigate strategies to solve problems involving addition and subtraction of fractions with the same denominator (VCMNA188)

Year 6: Solve problems involving addition and subtraction of fractions with the same or related denominators (VCMNA212)

Year 7:

- Solve problems involving addition and subtraction of fractions, including those with unrelated denominators (VCMNA243)

- Multiply and divide fractions and decimals using efficient written strategies and digital technologies (VCMNA244)

Connecting fractions and decimals

Year 4

Recognise that the place value system can be extended to tenths and hundredths. Make connections between fractions and decimal notation (VCMNA159)

Year 5

Recognise that the place value system can be extended beyond hundredths (VCMNA189)

Compare, order and represent decimals (VCMNA190)

Year 6

Make connections between equivalent fractions, decimals and percentages (VCMNA217)

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Thank you

Identifier first line

Second line